Phase errors in gossamer membrane primary objective gratings

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ABSTRACT

A ribbon-shaped primary objective grating (POG) telescope lends itself to deployment in space, because it can be stowed for transport on a roll. Unlike mirrors which need to be segmented for sizes beyond the diameter of the fairing or payload bay, the ribbon is a continuous integral surface transported on a drum and unfurled during deployment. A flat POG membrane abandons a standard three dimensional figure requirement of mirrors and solves the problem of making primary objectives from tensile structures. Moreover, POG telescopes enjoy relaxed surface dimensional tolerances compared with mirrors. We have demonstrated mathematically and empirically that the tolerance for flatness relaxes as the receiving angle increases toward grazing exodus where the magnification of the POG is greatest. At the same time, the tolerance for phase error is worsened as the angle of reconstruction moves toward grazing exodus. The problem will be aggravated by the rigors of the space deployment environment. We give a mathematical treatment for the flatness and phase error. We mention an engineering method that could ameliorate the error.

Keywords: gossamer, membrane, space, telescope, primary objective, grating, holography

1. INTRODUCTION

Surface area of the primary objective is precious in space telescopes. The very feature that makes space deployment so attractive, the lack of any atmospheric interference leading to extraordinary angular resolution, also makes it very difficult to launch, deploy and maintain large telescopes. Once in service, there are no gravitational limits on size, but gravity fights every gram that is intended for orbit. All of these constraints make gossamer membranes attractive as materials for primary objectives. They can be folded up, their mass is negligible, and the hope is that their cost will be low. A practical membrane primary objective has become a Holy Grail sought by astronomical instrument makers.

Many attempts have been made to produce a conventional mirror primary objective telescope in raw materials with gossamer membrane qualities. Balloon-like inflatables, truss-based stretched membranes and even faceted flats have received study. In all cases there have been errors in figure that render the large primary essentially less useful than conventional glass telescopes of smaller aperture.

In 2002 we proposed an entirely new concept in primary objective design that utilizes the dispersion principle to overcome many of the structural issues associated with a giant space telescope.[1] A primary objective grating (POG) is used instead of the mirror. The design principles are such that the radiation reaching the POG is dispersed to its side at an angle of grazing exodus where diffraction images can be collected by a much smaller conventional spectrographic telescope. We pictured a giant telescope as cargo stowed within a rocket's cylindrical faring. Our design presumes that a POG can be stowed as a roll plastic inside the delivery vehicle and unfurled as a ribbon after its arrival in space. The size of the POG is extensible to kilometer scale without a serious weight penalty. Unlike a mirror, the shape of the POG is flat. Obtaining a suitable optical figure could be a matter of mounting it on a stretcher. The process exploits the natural tensile forces that govern membrane architecture, i.e., membranes want to be flat.

A natural skepticism greeted our novel concept. First and foremost critics felt that it would difficult or even impossible to produce a figure at kilometer scale that would not be so completely uneven as to ruin the images the telescope was supposed to collect. Since its proposal in 2002 we have looked at the flatness parameter in terms of image formation by dispersion, and we can say that flatness tolerances are far less of a problem than was feared. On the other hand, phase errors in a diffraction grating are more serious.

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2. POG PRINCIPLES

The pose of a POG deployed in space must vary its incident angles to sequence the wave lengths of the acquired objects. When the POG has a pitch equal to the wave length of the radiation at its zenith, it will reconstruct its first-order for that wave length at angle of 90°. Of all types of gratings, these very high frequency gratings have the widest free spectral range, that is, the first-order has the most wavelengths where higher-orders do not overlap. Since these waves are collected near the edge of the grating, the geometry lends itself to flat telescope. The grating length can be many times greater than the secondary mirror that collects the dispersion from the grating. The geometry is shown in Figure 1.

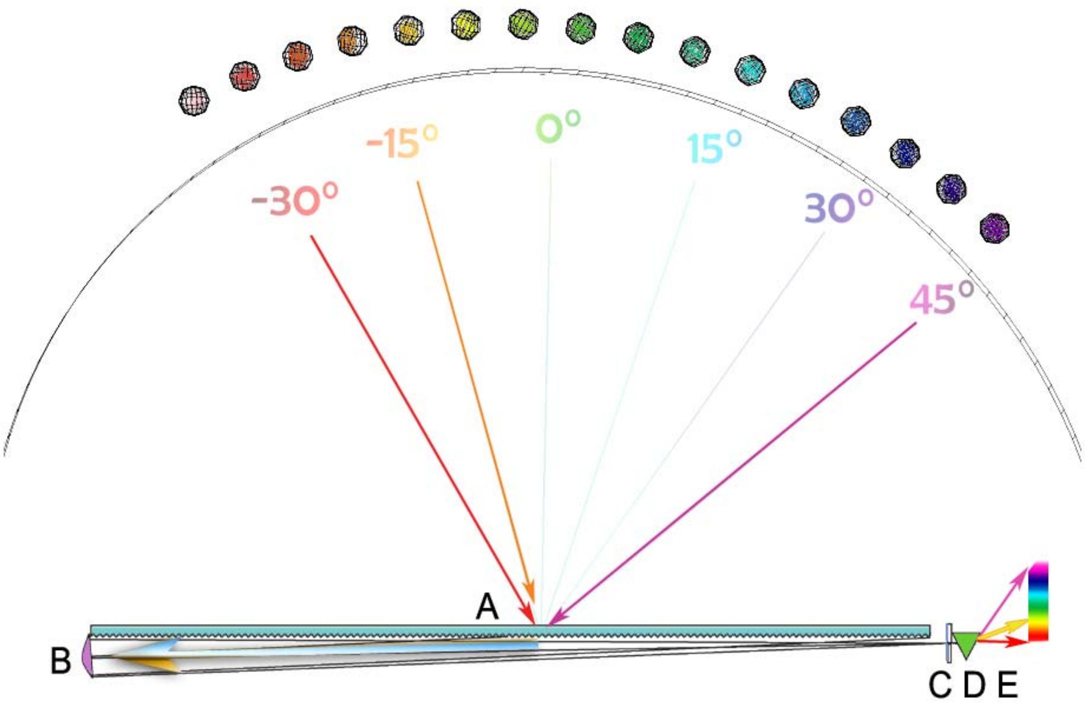


Fig. 1 POG **A** redirects light to secondary parabolic mirror **B** that focuses light on slit **C** of spectrograph **CDE**.

2.1 Space POG

Simple rotation analogous to a radar antenna will provide angular variation and also inertial stability. We call the embodiment "The Boloscope." Outboard secondary telescopes create the ballast. The inertial forces provide tension for the membrane. The Boloscope has inertial characteristics anticipated by space tethers. The inertial forces can be modeled using software for space tethers developed by Rob Hoyt.[2]

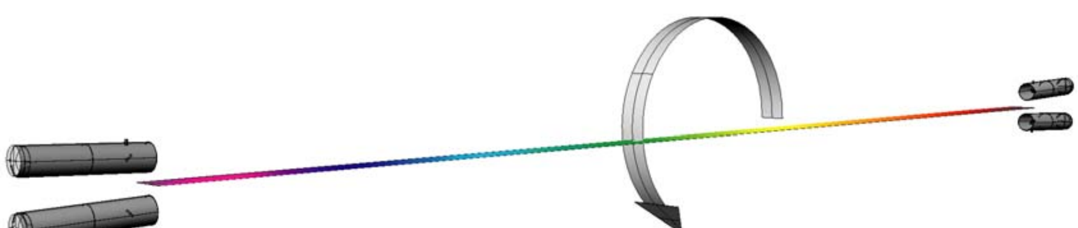


Fig. 2 "The Boloscope," a rotating platform, illustrated with symmetrical secondaries

To add the intrinsic stiffness of a curved surface while also concentrating flux in the narrow dimension, one option is to create a parabolic trough. The superstructure bears similarity to the Dual Anamorphic Reflector Telescope (DART).[3]

The DART was considered for sub-millimeter and deep IR, because it was not expected to hold a figure suitable for visible light, but the trough might have supported a figure suitable for a POG. A rigid embodiment would have less need for the Bolo's centrifugal force stabilization. It could enjoy a stationary pose where changes in incident angle were imparted by a gradual orbital rotation around a gravitational center such as the earth, the moon or the sun. This would result in longer integration times and the possibility of narrow bandwidth scanning.

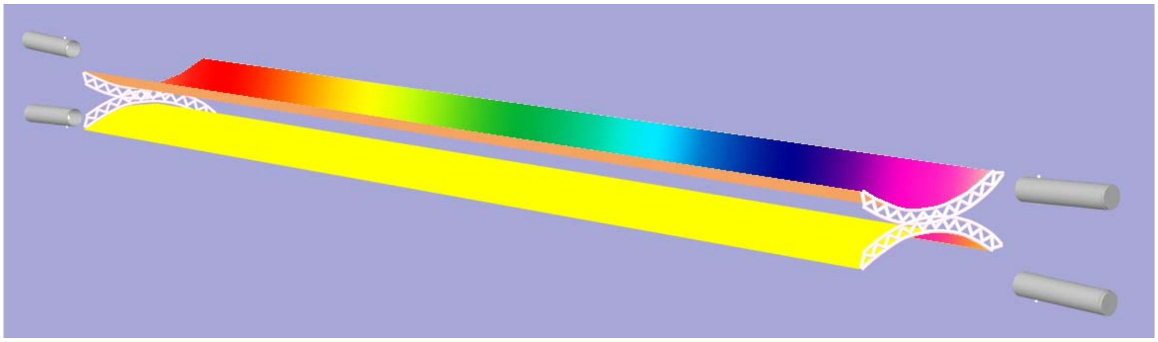


Fig. 3 A dual parabolic truss superstructure for rigidity

While it is true that such geometric optics belie the difficulty of making the physical optics workable, we have addressed some of these issues in other disclosures. If the reader will allow that such architecture can be made to work, there is a marked difference in the paradigm of physical plant needed when compared to traditional telescopes.

Locations for a POG space telescope could include low earth orbit (LEO) and geostationary earth orbit (GEO). Their selection might evolve in that order over a development period leading to the expertise needed for the LaGrange points.

A LEO tin can experiment would be useful simply to evaluate the POG concept. In full scale, LEO is the most likely station for astronaut service calls. The Hubble Space Telescope proved the value of such an option. In LEO, gravitational attraction toward the center of the earth could be exploited to maintain a vertical pose. This is a relatively static posture that would also simplify docking. Incident angles on the firmament would vary in the same manner as the ground-based version of the POG instrument.

Large primary objective mirrors on the scale of 100 m have been contemplated for GEO insertion as reconnaissance telescopes. Repeating history with Keyhole for reconnaissance and Hubble for astronomy, if the former can be achieved, then the latter will follow. A geostationary reconnaissance telescope would require the large aperture to have meaningful ground resolution, and a POG might prove more practical than the robotic self-assembly and maintenance of a gigantic 100 m mirror, the baseline for a geostationary reconnaissance satellite.

A hybrid option, diagrammed in Figure 4, would place a secondary on the ground as a static fixed installation for a POG at GEO. Under this regime, the secondary could be serviced on the ground with relative ease. The scale of the ground-based secondary collector would be unprecedented, in the 100 m class, but as a zenith tube it could rest flatly on a firm foundation. The GEO POG might be on the order of 1 km x 20 m. The entire instrument could have an optical diffraction limit on the order of 0.001 arcsecond, as is suitable of exoplanet discovery, if adaptive optics were capable of neutralizing the atmosphere for such an enormous secondary mirror aperture on the earth's surface.

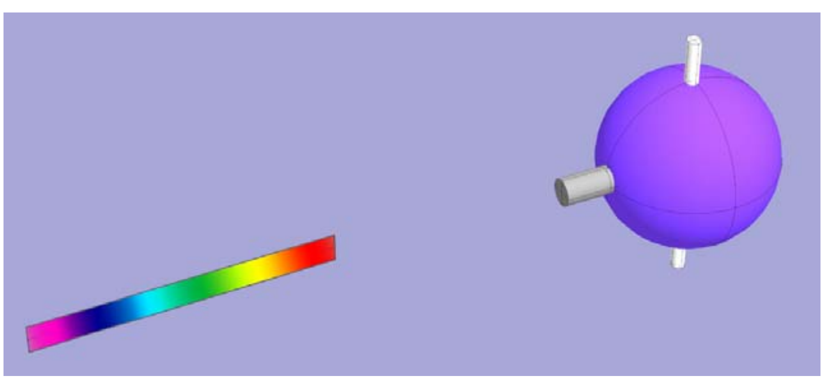


Fig. 4 GEO POG at geostationary and secondary on equator (not to scale)

3. TOLERANCES

3.1 Flatness tolerance of a membrane

For a ribbon POG, diffraction occurs along the long axis, while the shorter width is no different than a mirror or refractive flat plate, depending on whether to POG is used in reflective or transmission mode, Figure 5.

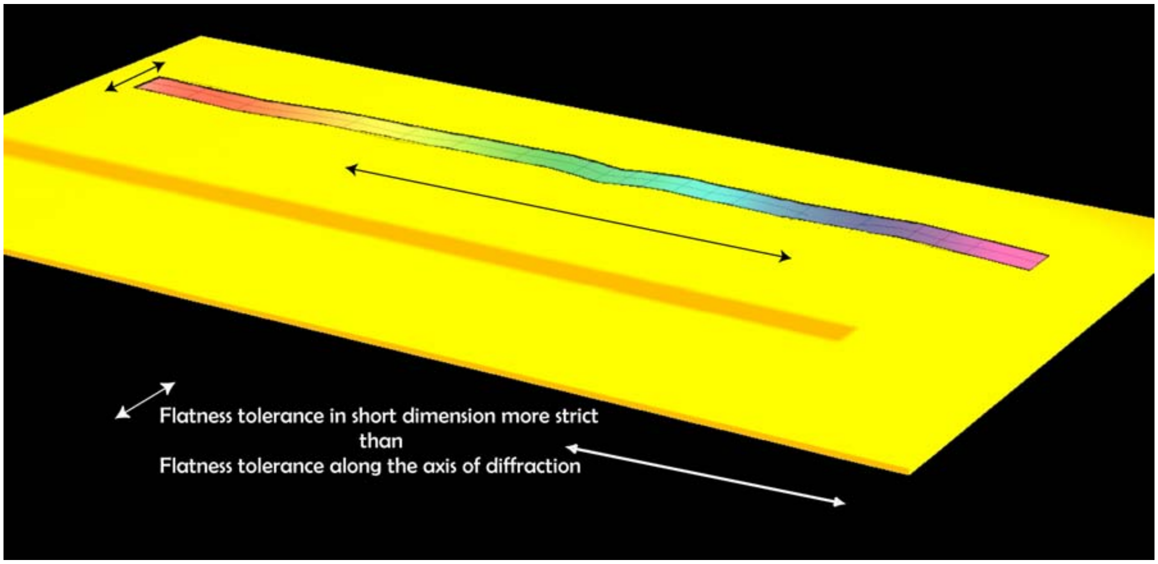


Fig. 5 Flatness specification differs markedly between the long diffraction axis and the short reflection or refraction axis

Two sets of equations are required to establish tolerances. We have investigated the flatness tolerances for the diffraction grating using a model that considers the effect of displacement over a considered length. Transmission and reflection flat tolerances we presume are to be found in the literature. We have limited our present investigation to the physical optics that are not in the literature.

As a result of our study [4] we now assert that the tolerance for flatness in the diffraction dimension relaxes as the angle of grazing increases. This is because the diffraction image is formed by the constructive sum of in-phase waves originating from all grating grooves. When the light exits along the grating axis, variations in grating height have a negligible effect on the phase of the waves passing nearly parallel to the grating surface and exiting to the side.

We can approximate the flatness tolerance for grazing exodus by using a displacement *d* over considerable length *L*. See Figure 6. We compare two rays that are rotated by angle $\Delta\alpha$.

$$\Delta\alpha = \arctan\left[\frac{d}{L}\right] \tag{1}$$

Angles of incidence i_1 and i_2 are rotated by $\Delta\alpha$ as are receiving angles r_1 and r_2. Assuming incidence and receiving angles are known, we can compare the change in wavelength as a function of the surface flatness by calculating two wave lengths. $\Delta\lambda = \Delta\lambda_1 - \Delta\lambda_2$ where $\Delta\lambda$ is the spectral resolution and λ is determined by the Diffraction Equation:

$$\lambda = \frac{(\sin(i) + \sin(r))}{n} p \tag{2}$$

where:

λ = wavelength of radiation
i = incident angle
r = receiving angle
p = grating period
n = diffraction order

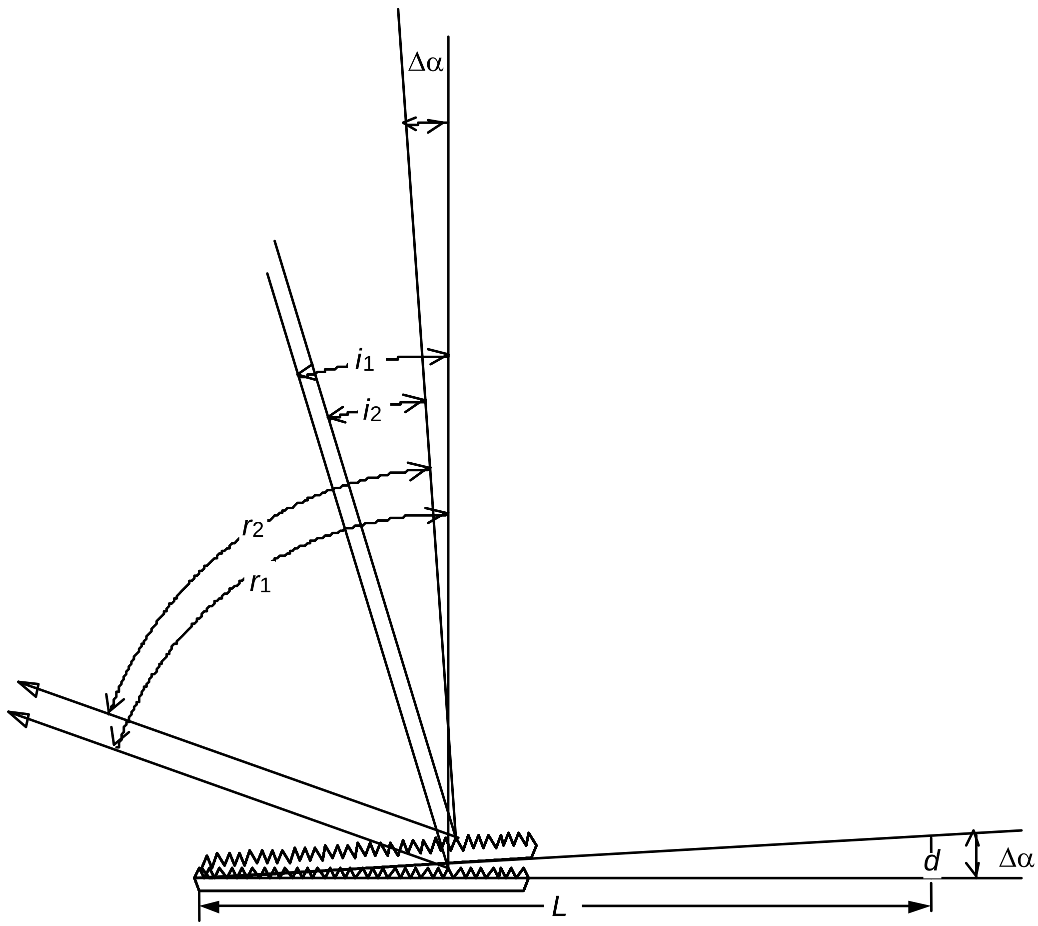


Fig. 6 Parameterization of flatness tolerance specification

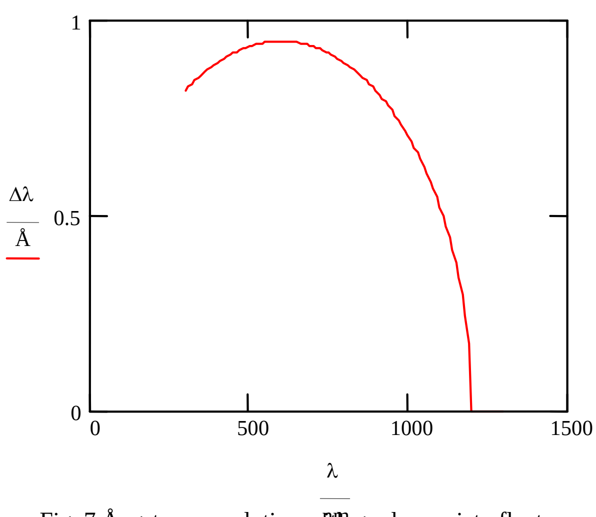


Fig. 7 Ångstrom resolution with garden variety float glass rated at 8 waves per inch supporting a 600 nm grating configured at grazing exodus *r* = 88°.

We are examining incident radiation arriving from near the zenith. This analysis suggests that when flatness is held to the same tolerance as a state of the art mirror, e.g., 100 nm over 2.5 m, the limit of resolution of an equivalent POG for radiation from the zenith is 0.000025 nm. When the light is at both grazing incidence and grazing exodus, there is no phase delay regardless of surface irregularity, and $\Delta\lambda \rightarrow 0$.

Forgiving surface tolerance is appreciated by diffraction grating manufacturers who use float glass as a substrate where Ångstrom resolution is specified. In Figure 7 we plot a relationship of λ vs. Δλ where the substrate is a garden variety float glass. In the example, exodus angle *r* = 88°.

To correlate spectral resolution to angular resolution, we look at the linkage between the two. Small changes in wavelength correspond to small changes in the angle of incidence when the receiving angle *r* is fixed. We can determine Δ*i* by taking the difference of the change in λ over one slope direction in the curve. We use Equation. (3).

$$\Delta i = \arcsin\left[\frac{\lambda}{p} - \sin(r)\right] - \arcsin\left[\frac{\lambda - \Delta\lambda}{p} - \arcsin(r)\right] \tag{3}$$

Under the presumption of a high frequency grating where the acquisition is exclusively in the first-order, we set $n = 1$ and exclude that term.

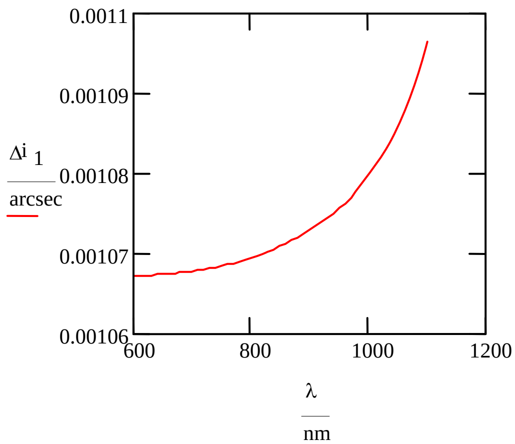


Fig. 8 Angular resolution for 600 nm flatness tolerance

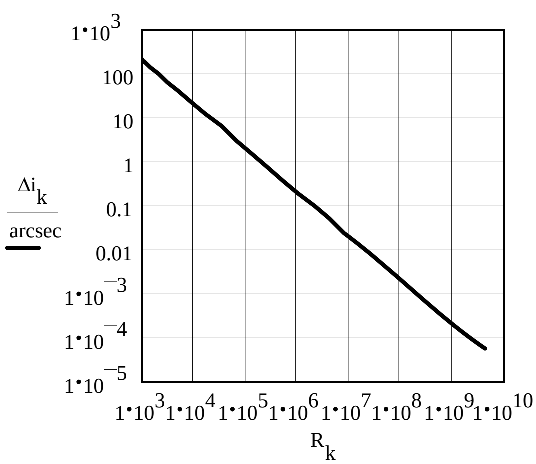


Fig. 9 Δi as a function of R using a log scale

Using Equation (3) we consider another case using a tighter tolerance for grating flatness of 600 nm over 100 m. This tolerance would be considered substandard for mirror primaries, but it represents a goal for very large membrane structures. The POG error due to flatness is on the order of 0.001 arcsecond, Figure 8. This approximates the angular resolution that will separate an exoplanet from its host star. The theoretical limit for Δi for a grating as measured along the diffraction axis for a POG of 100 m length can also be calculated from the resolving power R on the basis of the grating length L. $R = L/\lambda$. $\Delta\lambda$ can then be known from $\Delta\lambda = \lambda/R$.

For 600 nm infrared radiation incident along a 100 m grating, $R = 1.667 * 10^8$. The resolving power limit of $\Delta\lambda$ is $3.6 * 10^{-5}$ Å. With Equation (3) we can determine a theoretical limit of Δi. For a 100 m POG, $p = 600$ nm, we calculate that $\Delta i = 1.245$ milliarcsecond.

We show the generalized relationship at this wavelength for a wider set of first-order resolving powers in Figure 9. The evidence suggests that if the POG can be kept flat end-to-end to a tolerance that falls within the wavelength of the light it is measuring, the primary objective can achieve a precision of angular resolution performance that is close to the theoretical limit of its resolving power as a diffraction grating.

If this is true, the flatness tolerances for diffraction gratings are different than figure tolerances for mirrors. Designing a space borne POG presents relaxed tolerances for its length as compared with its considerably narrower width. In the lateral (short) dimension, the POG is essentially the same as a mirror, and the requisite flatness tolerance is well understood from the simple relationship that the angle of incidence equals the angle of reflection. We leave this analysis for later, although the physical realization of a large mirror flat may represent a greater challenge than the POG. Optical flats have been made in membrane materials that reach surface roughness of 20 nm and overall flatness on the 100 nm scale. The cited demonstration was made on a 0.75 meter scale experiment. Extensibility to larger size would seem unimpeded. The underlying physical principle is that tensile membranes want to be flat.

Experiment confirms the forgiving flatness tolerance of diffraction gratings at angles of grazing exodus, Figure 10.

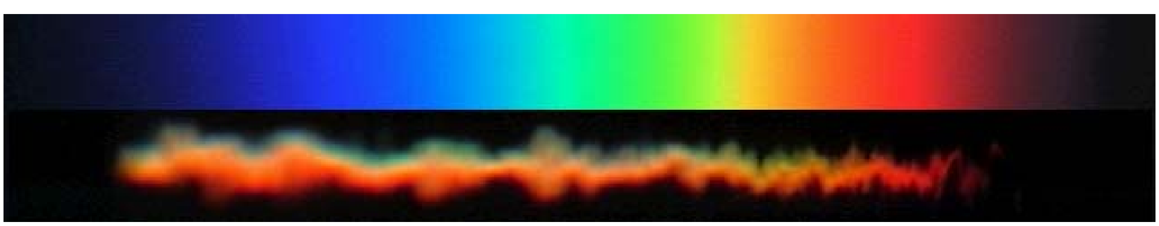
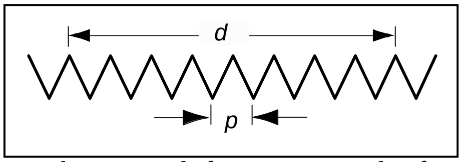


Fig. 10 Comparison of dispersion (above) and reflection (below) in the exact same embossed reflection grating

3.2 Phase tolerance

The peak to peak groove phase relationship presents another tolerance problem. Even if the phase error is minimal at manufacture, it can easily be aggravated by environmental conditions after deployment in space. A tensile structure will face stress factors that are both uniform and non-uniform, creating errors, some continuous and others stochastic.

Reworking Equation 3 to make spacing between rules Δ*d* a variable we set a dependency for Δ*i*.



$$\Delta i = \arcsin\left[\frac{\lambda}{d-\Delta d} - \sin(r)\right] - \arcsin\left[\frac{\lambda}{d} - \arcsin(r)\right] \qquad (4)$$

We show a result for grazing exodus for a 600 nm pitch grating in Figure 11 (left). If a grating had only two adjacent grooves, milliarcsecond angular resolution would presume pitch accuracy to approximately the diameter of a proton. However, the resolving power of a diffraction grating is directly proportional to its length. In Figure 11 (right) we show the phase error for angles of Δ*i* by setting d to 1 cm. Now Δ*d* is measure in 100's of nm rather than thousandths of Å. We do note the tolerance is less forgiving for angles of grazing exodus.

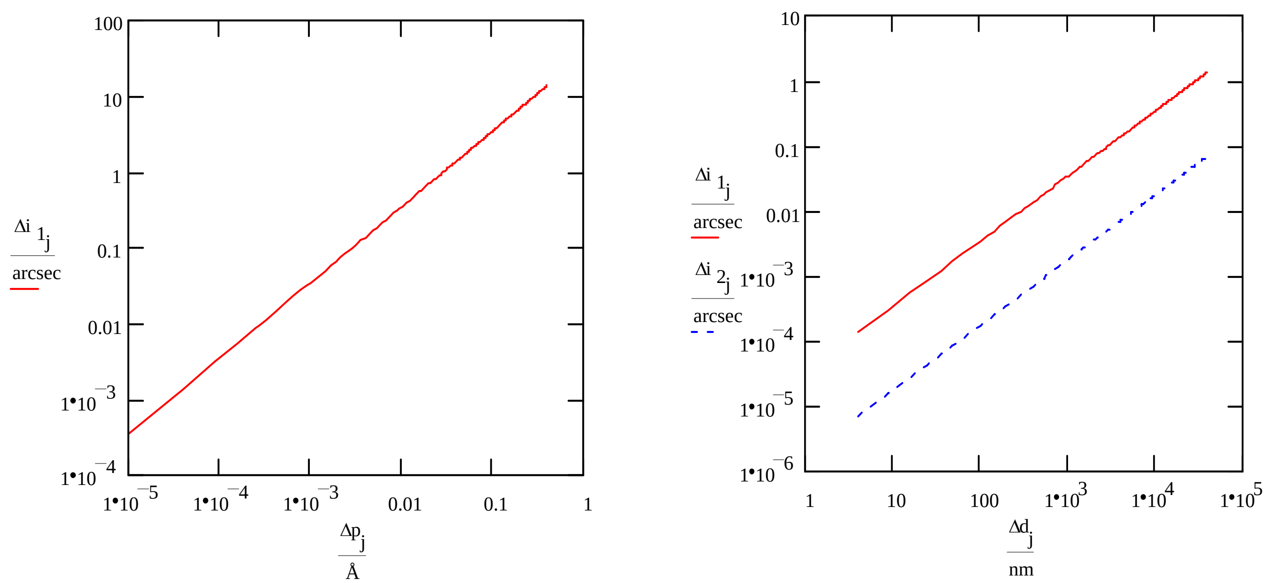


Fig. 11 (left) Phase tolerance for interference from two grooves is daunting compared to grating phase tolerances (right)

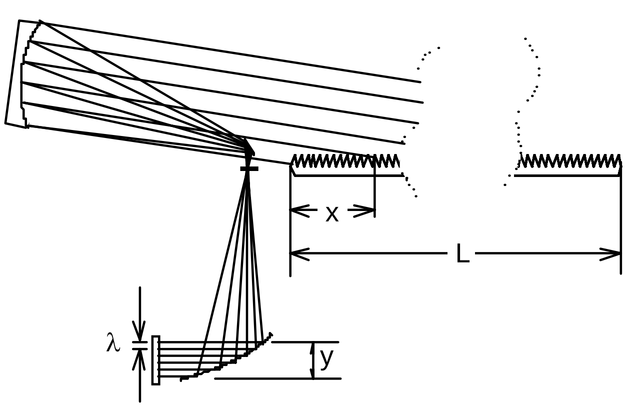


Fig. 12 Phase coherence model

We made a preliminary analysis that may be heuristically consistent with the result shown in Fig.11 (right) where the phase problem appears to be aggravated by greater angles of *r*. While a thorough analysis for this problem has yet to be done, in a prior publication we suggested that a coherence length be taken from the sub-section of a grating required to produce a specific wavelength in the receiver as a fraction of the entire POG length. Ray paths to the collector can be separated from each other in single wavelength divisions if the secondary has considerable width. If the secondary is modeled with the considerable length *y*, any wavelength segment λ can be mapped back to the grating of length *L* to determine the incremental steps *x* over which phase error must be within tolerance. The relationship is $x = L\lambda/y$.

This relationship is shown in a cross section in Figure 12. This may explain the tighter tolerances required as r approaches grazing exodus. More study is required. We do suspect that an active substrate that can vary its tensile properties (such as the “muscle” membrane investigated contemporaneously with our present research by NIAC Fellow, Joseph Ritter) will become mandatory to buffer the gossamer membrane against stretching and displacements that affect uniform grating pitch.

4. CONCLUSION

The ribbon-shaped POG lends itself to deployment in space, because it can be stowed as a gossamer membrane transported on a roll. The configuration for stowage conforms to the payload geometry of delivery vehicles. Unlike mirrors which need to be segmented for sizes beyond the diameter of the fairing or payload bay, the ribbon is stored as a continuous integral surface on a drum and is unfurled during deployment, Figure 13.

Diffraction addresses the conundrum of making a collector from tensile material. We suggest abandoning the three dimensional figure that has been so elusive in all prior attempts to design a gossamer membrane primary objective. A POG is flat. It also turns out that gratings differ significantly from mirrors with regard to their operational flatness figure tolerances. Long live the difference.

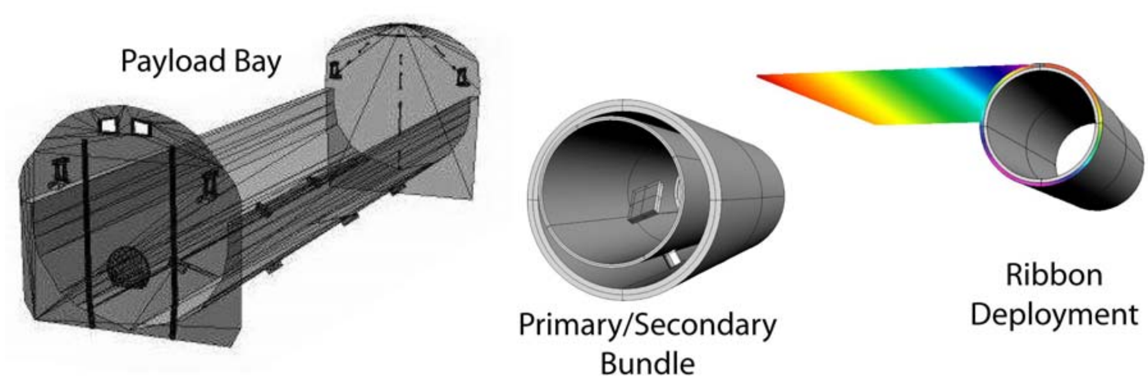


Fig. 13 A cylindrical payload is compatible with the stowage of an extensible gossamer membrane POG and a secondary parabolic mirror that is modeled on a conventional astronomical telescope with a spectrograph.

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