Conservation of étendue in a primary objective grating telescope

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ABSTRACT

A novel telescope design that uses a primary objective grating has been shown to have valuable performance features for multiple object spectroscopy as well as a flat collector with relaxed figure tolerances suitable for membrane substrates. The commonly applied figure of merit called étendue takes on new meaning here when compared to telescopes which employ a parabolic mirror In this new telescope design, a mirror is in the secondary. One dimension of the mirror conforms to the familiar metrics for measuring field-of-view and surface area. However, in the other, an anamorphic magnification feature of plane gratings takes effect at angles of diffraction as subtended at grazing off the grating plane normal. When the secondary parabolic mirror is placed at a grazing angle, compression increases exponentially, permitting unprecedented fields-of-view, collection surface areas and aperture in one of the two dimensions. We provide an analysis that calculates the compression, resolution, and field-of-view. More importantly, we study how these parameters are conserved by a secondary parabolic mirror notwithstanding that the mirror diameter can be significantly less than the length of the primary objective grating. Zemax models are used to show that along the dimension of the grating, étendue is conserved in this telescope architecture.

Keywords: Ètendue, Primary Objective Grating, Multiple Object Spectrograph,

1. INTRODUCTION

A multiple object spectrographic telescope has been proposed which consists of a traditional spectrographic telescope set at an angle of grazing exodus relative to a primary objective grating (POG).[1] Hereinafter we refer to this novel telescope as THE MOST for The High Étendue Multiple Object Spectrographic Telescope. In this paper we consider how angular resolution of the POG is conserved by the considerably smaller parabolic mirror in the secondary.

1.1 THE MOST

If a plane diffraction grating disperses to a fixed receiving angle r then over increments in the angle of incidence i the resolved wavelength $\Delta\lambda$ will vary proportionally with a change in incidence Δi. The relationships are shown in Figure 1.

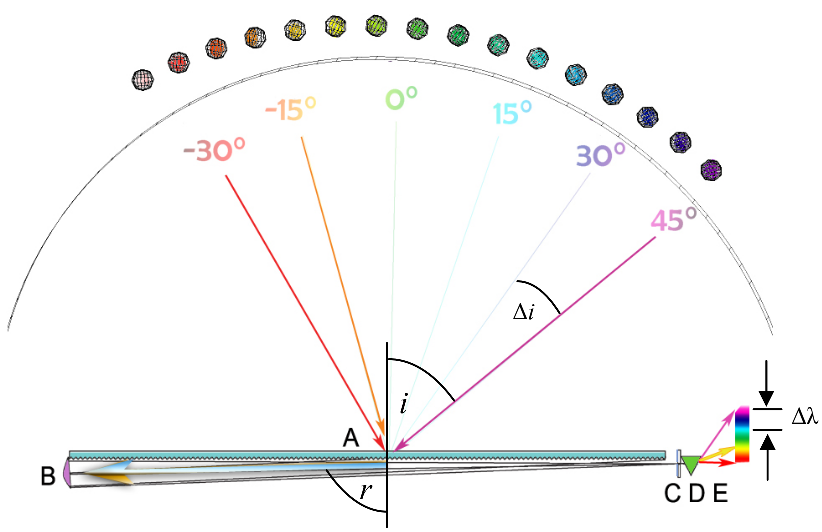

Fig. 1 A star at variable angle i is diffracted by grating **A** at fixed angle r to secondary parabolic mirror **B**, and the light is focused on slit **C**. A secondary disperser, **D** separates out all the visible objects as spectrum **E**. Three distinct targets at angles i = -30°, -15°and 45° are illustrated in bold lines.. In the course of an observation cycle, spectrograms are taken over all wavelengths $\Delta\lambda$ for all stars.

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1.2 Free Spectral Range

THE MOST has a field-of-view equal to the arc on the sky where it projects a spectrum without interference from competing diffraction orders. To maximize this angle, we select the first-order to enjoy the widest dispersion. The free spectral range is nominally 0 to 90°. Below 0° the second-order overlaps the first-order at shared wavelengths, creating ambiguities. We illustrate the relationships in Figure 2. A 90° field-of-view is unheard of in an astronomical telescope.

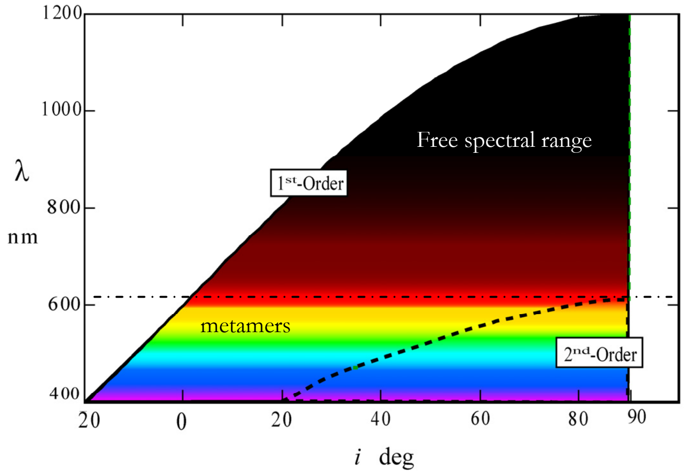

Fig. 2 For a POG with a groove pitch of 600 nm, the free spectral range rungs from 620 nm to 1200 nm when $r = 89°$.

Modifications can be made to the basic design to further open up the field-of-view. If a second mirror faces the POG from the opposite direction and an occluding baffle is employed, Figure 3, then a shadow is cast over the competing second-order. Where this shadow occurs, the first-order on one side is unambiguous. Moreover, the second mirror provides most of the other half of the sky at the same time. The field-of-view is an arc effectively across a line of right ascension, nearly 180° as shown in Figure 4. The gamut of wavelengths is wider than double the grating pitch.

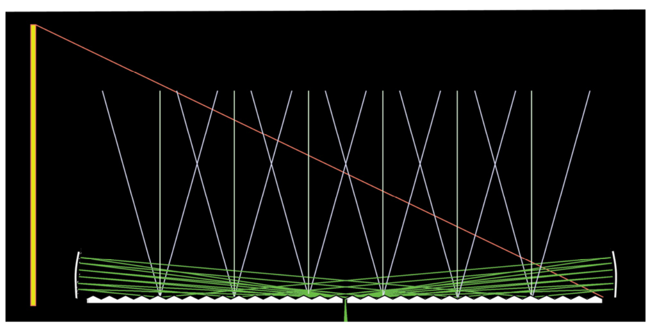

Fig. 3 Zemax model of THE MOST with dual secondary parabolic mirrors and a left-hand baffle to partially occlude the second-order

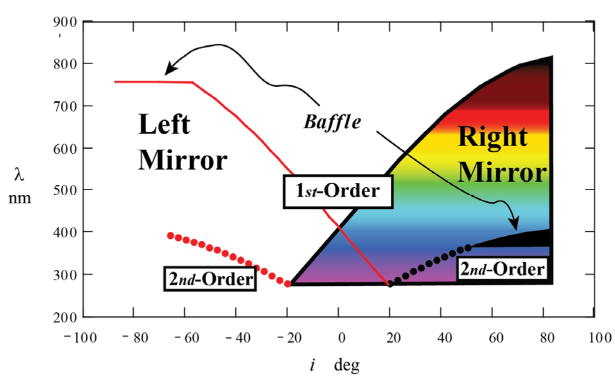

Fig. 4 Combined effect of dual mirrors and baffles on available spectral range of a POG with grating groove pitch of 400 nm

1.3 Collection area

THE MOST, exploits a grazing exodus configuration that allows the POG to potentially enjoy maximized lengths. In space deployment using a 2.4 m secondary mirror at an angle of r = 88.5°, POG length can exceed 100 m. On the ground, kilometer scale is conceivable for this type of optic, because the secondary mirror might be d = 20 m at grazing angles r = 88.8°. See Figure 5. The collection areas are the product of the mirror diameter and the POG length, Figure 6.

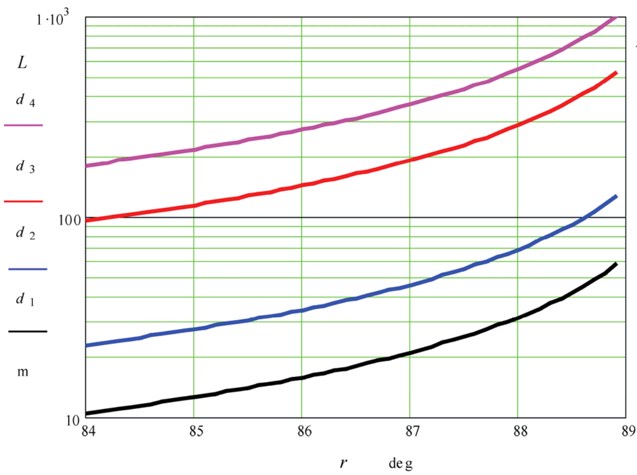

Fig. 5 Maximum POG lengths L as a function of receiving angle r for secondary mirror diameters d_n of 1.1, 2.4, 10 & 19

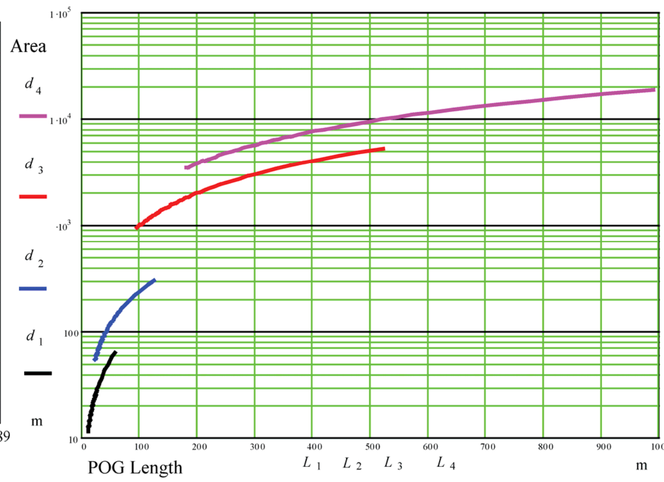

Fig. 6 POG collection areas A as a function of the four secondary mirror diameters d_n and lengths L of Fig. 5

1.4 Magnification

Grazing controls the magnification in the ratio of L/d. The logarithmic scale of ordinate values in the graphs of Figures 5 and 6 are indicative of leverage that takes place as the angle of diffraction increases toward its limit of 90° where the POG could theoretically stretch off to an infinite length. There is a reciprocal of aperture a over angles of incidence from the zenith to the horizon. The change in magnification over the line of right ascension decreases exponentially as sources move toward the horizon. The relationship is shown in Figure 7 for three static receiving angles at grazing exodus. At the horizon, the entry to the POG is along the grating plane, and the resultant aperture is zero. At the zenith the aperture is at its maximum as a plane wave strikes the POG with the broadest wavefront. We consider i = 0° as the most stringent case test for conservation of étendue, because it places the greatest demand on the resolving power of the secondary.

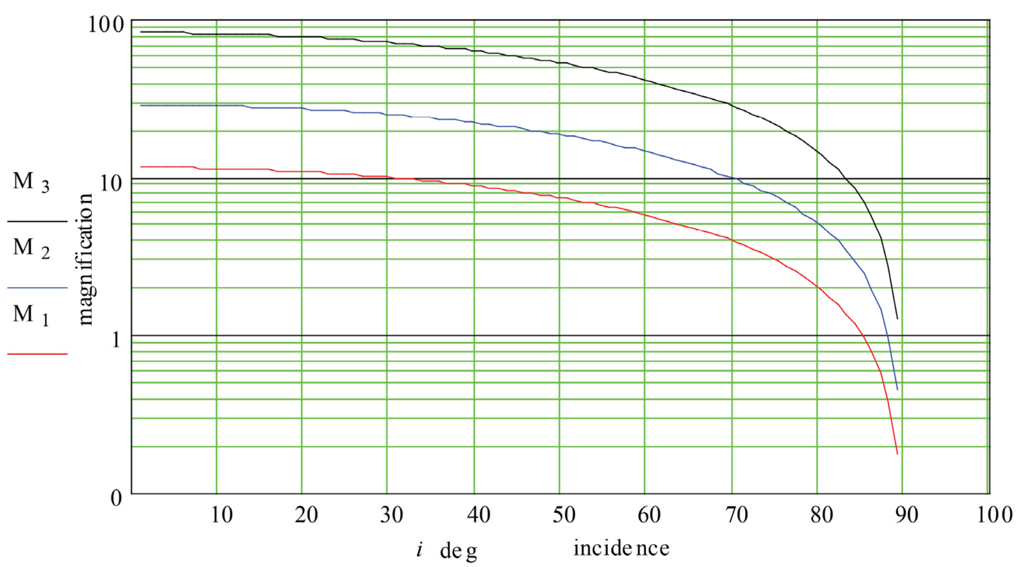

Fig. 7 Magnification a/A over the line of right ascension incident upon the POG for grazing angles r = (85°, 87°, 89°)

2. ÉTENDUE

Survey telescope performance is rated by the product of the collection area and the field-of-view. The term étendue is used to specify this figure of merit. Étendue is symbolized as AΩ where A is the collection area and Ω is nominally the square ω, the field angle, or ω^2

2.1 AΩ of survey telescopes with mirror primary objectives

There is a survey telescope being installed where AΩ exceeds 300, the Large Synoptic Survey Telescope (LSST) which enjoys a field-of-view of 3.5° in each axis. It has a primary mirror diameter of 8.3 m. The primary diameter does not fully describe the collection area because of an unusually large 3.4 meter center hole. The hole facilitates a triple conjugate f/1.23 effective focal length that gives LSST a wide field-of-view based on a Paul-Baker/Mersenne-Schmidt architecture. Given the central hole, the primary objective collection area is nominally 35 m^2. Nonetheless, with the trade for collection area, the LSST enjoys an AΩ of approximately 320, many times greater than any prior art. It is presently under construction in Chile and when completed will have the greatest étendue of any astronomical telescope in service. By a process of rapid scanning, it can assemble a mosaic of the entire southern sky in the course of several nights.

LSST has an angular resolution at seeing, somewhat less than the theoretical limit for 8.3 m apertures. However, adaptive optics (AO) cannot be used with high AΩ ground telescopes, lowering the bar for angular resolution. Another limitation is that the telescope has no spectrometer. There are six filters that bracket wavelength bands from 320-1060 nm. Filter exchange is required to obtain any one wavelength band.

By way of comparison with LSST, an agile survey telescope already in service is Panoramic Survey Telescope and Rapid Response System (Pan-STARRS) which has an AΩ of 60. Its primary objective is 3.6 m and it has a field-of-view of 7 deg^2.

2.2 THE MOST's AΩ

In the case of THE MOST, the fields-of-view are asymmetrical across two axes. One axis is the free spectral range of 90° as described in Section 1.2, above. As shown in Section 1.4, the useful portion of the arc declines as magnification tapers off. On the other hand, if a second mirror on the opposite side of the POG is used as per Figure 3, the pair of fields-of-view could easily sum to 100°. The other dimension has a field-of-view equal to the parabolic mirrors in the secondary. One may reasonably anticipate 1° field-of-view for that optical axis, so $\Omega = (\omega_1 \cdot \omega_2) \cong 100\ \mathrm{deg}^2$.

A prospective embodiment being studied for THE MOST is a kilometer scale POG and a secondary parabolic mirror of 20 meters or A = 20,000. Given a field-of-view in excess of 100° x 1°, the resulting arithmetic is: $A\Omega = 2 \cdot 10^6$. By that measure, THE MOST's étendue would be nearly three orders of magnitude greater than the LSST.

2.3 Concerns about conservation of étendue in POG telescope

The value of a survey telescope depends not on the very simple arithmetic of AΩ. It should include the effective angular resolving power of the instrument as well as overall throughput of flux. The LSST is a telescope designed to work at seeing rather than at the theoretical limit of the primary objective's diameter > 8 meters. There are trades in resolution for wide angle through a triple conjugate, so the large number given for AΩ must be taken with "a grain of salt."

Similar reservations and worse have been expressed about the survey capability of the POG architecture in THE MOST. When reviewed by an expert panel at NASA, the design was deemed by some to be fatally flawed, because it was thought that the diffraction limit of the secondary mirror would control the resolving power at the image plane.

> The fatal flaw lies in the... claim, that the resolution will be high. This particular error has been made before and will doubtless be made again. They argue that light falling on the grating at normal incidence and being observed in a diffracted order at "grazing exodus" will be diffraction limited by the length of the grating (L) at λ/L. In reality, the diffraction limit will be $\lambda/(L \sin(\phi))$ where ϕ is the angle between the surface of the grating and the direction of the diffracted order. So the diffraction limit is set by the diameter of the parabolic collector in both dimensions. Consider an aperture of diameter d with light of a single wavelength incident upon it at the normal. The Fresnel equation tells us that the resolution of the emerging light will be λ/d because the wavefront is uniform across the aperture. Now bring a wavefront at grazing incidence (ϕ small) onto a mirror of width d and length $L=d/\sin(\phi)$. After the reflection, allow the light to move away from the mirror, and it will, to first order, resemble a square beam of size dxd. This is identical to the first mentioned aperture of diameter d, which

we know to have resolution of λ/d. This has been recognized and measured in the lab many times for grazing incidence optics. Now bring light at normal incidence onto the grating. The grazing exodus of certain wavelengths is created at the angle at which the light from each point on the grating comes into constructive interference. That is, at a large distance from the grating, all the wavelets will be moving the same direction at the same phase. So view the grating at the exodus angle, and the wavefront of the diffracted light will have uniform phase across an aperture of d. The resolution will be λ/d, not λ/L .[2]

Admittedly, there could be a fatal flaw overlooked by the inventor, but a tendency to conflate all grazing optics has penetrated discussions ever since this POG architecture was first introduced twelve years ago. The adjective “grazing” can trigger knowledgeable responses from astronomers based on telescopes and spectrographs that are highly researched where incident and receiving angles are at grazing. However, it cannot be assumed that the limits in resolution imposed on secondary spectrometers by their focusing mirrors automatically apply here. The POG and its focusing mirror are in the primary before the slit. They are not a secondary spectrometer after the slit. We will show that the mirror in question focuses onto a slit that may vary in width inversely with the diameter of the mirror. Resolving power as the mirror decreases in diameter is matched by an allowable widening of the slit.

2.4 An analysis of conservation of étendue in POG telescope

When the Diffraction Equation is used in the analysis, the rules governing conservation of étendue can be seen to extend to a POG telescope over its broad spectrum.

THE MOST uses only the first diffraction order, so for any wavelength λ:

$$\sin(\mathrm{r}) + \sin(z) = \lambda/p \tag{1}$$

where p is the grating pitch, z is a star at the zenith and $r = 90°$ is at the grating plane.

Taking derivatives:

$$\frac{dz}{dr} = \frac{\cos(r)}{\cos(z)} \tag{2}$$

The ratio d/L is the compression of the beam where d is the diameter of the secondary mirror collecting light from grating length L. The resolution of the secondary mirror is understood to be limited by diffraction at $\Delta r = \lambda/d$, so resolution Δz at the zenith is:

$$\Delta z = \Delta r \frac{dz}{dr} = \frac{\lambda}{L} \tag{3}$$

It cannot be disputed that spectral resolving power is proportional to grating length L. That dependency is widely derived in the literature.[3] Practice in spectrograph design has always been guided by the understanding that an increase grating length can increase spectrograph resolution.

In the case of the first-order of diffraction, as employed by THE MOST, spectral resolution is determined by the number of grooves in the grating, a number set by length of the grating. Each grating groove is a point source radiator that reconstructs a tiny portion of an incident plane wavefront into a new spherical wavefront originating at that groove. The constructive interference these spherical waves combine into new plane waves exiting the grating. The parameter we symbolize as angle r represents not only a receiving angle but also the angle of reconstruction.

At reconstruction angles of grazing exodus, aggregate spherical wavefronts are compressed more closely together, creating more sharply defined plane waves. The maximum spectral resolution of a plane diffraction grating is at grazing. No one less than A. A. Michelson noted this in a 1927 publication where he wrote, “The maximum resolving power is attained for grazing incidence and diffraction; but in this case the intensity would be vanishingly small.” [4]

We acknowledge, as Michelson’s caveat suggests, that at grazing incidence the flux collected is proportional to the aperture facing to a source. We show the losses for a POG in Figure 7. At grazing incidence the aperture and its resulting magnification approaches zero as incident angle i approaches 90°. However, we presume that since a telescope is a flux magnifier it will present the widest possible apertures, i.e., those near $i = 0°$. We are not proposing a telescope that works at grazing incidence like a Wolter x-ray telescope. Its analysis developed for a grazing incidence device is not applicable to conservation of angular resolution by the grazing exodus optics of THE MOST. A grazing incidence analysis is applicable to a related diffraction microscope which we have developed on a separate track.[5] but it is not the topic here.

The question addressed here is whether the resolution of the focusing mirror in the secondary of THE MOST keeps up with increasing resolution of the primary as the angle of diffraction r approaches 90°. Our analysis is premised on comparison of slit size as a function of the reconstruction angle. It turns out that while diameters of secondary mirrors decrease with increased grazing angles, the requisite slit sizes needed to conserve étendue grow wider in direct proportion. In other words, the diffraction limit of the secondary mirror is allowed to decline, because wavelengths are proportionately magnified and may pass through larger slits.

We show this using a Zemax model of THE MOST, Figure 8, up to the slit, **C**, of Figure 1. In Figure 8 we see rays from the sky spanning 30°. The rays are colored by wavelength with red diffracted more than green or blue. We will be concerned with the ray from the zenith which is colored green. Magnification is greatest at this angle of incidence.

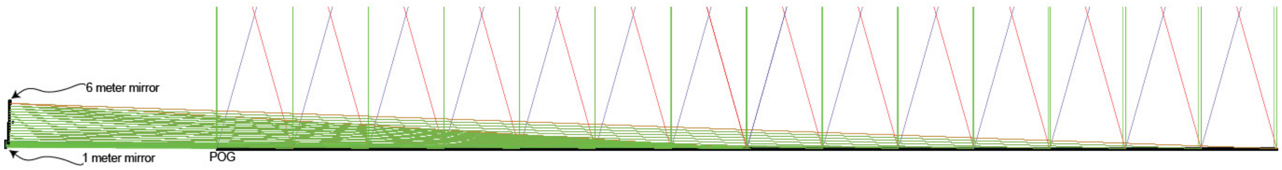

Figure 8 A Zemax model of THE MOST showing rays from sky diffracting off a POG to secondary mirrors on the left.

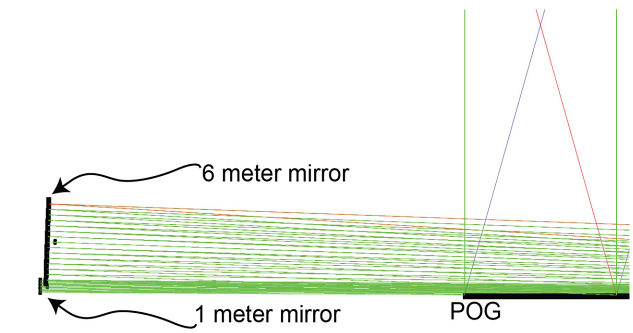

Fig. 9 Detail showing the pair of secondary mirrors in the model

In Figure 9 we see a detail from Figure 8 showing two mirrors of diameters of 1 and 6 meters at r = 89.6712° and 87.96315°. They have been positioned at angles of grazing exodus such that they both intercept the diffracted rays over the entirety of the same POG. In other words, POG length is identical in both cases.

The argument we are facing is whether equal POG lengths can be equally resolved by mirrors of different diameters. This POG model is 300 meters long. The fatal flaw would appear if the smaller mirror's wider point spread function (PSF) as predicted by the limiting factor of λ/d failed to produce a spectrum of equal resolution and Δz as the wider diameter mirror.

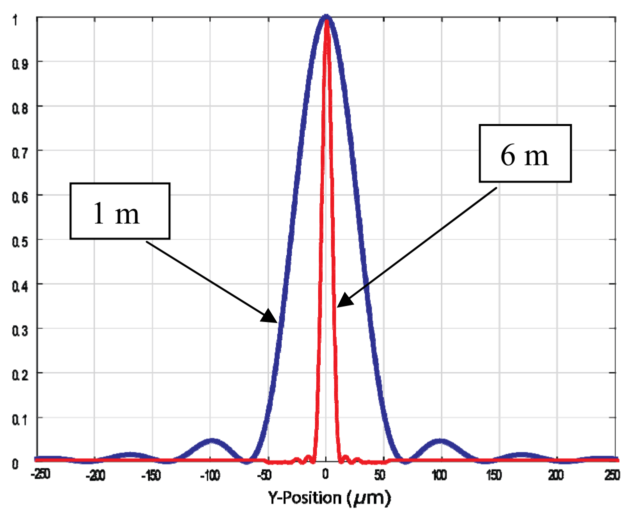

Fig. 10 Cross sectional Huygens PSF's for 1 m (blue) and 6 m (red) parabolic reflectors in a Zemax model of THE MOST. These ideal mirrors have Strehl ratio of 1 Wavelength is 550 nm.

The mirrors do have different PSF's as shown in Figure 10. As expected, the FWHM of the larger mirror is less than the smaller mirror. This compels the selection of the corresponding slits to have acceptance windows of different sizes in order to achieve the maximum spectral resolution offered by the POG.

The guidance we will use to select the corresponding slit widths is that the mirrors must resolve a point to the base of the PSF. This means that for the smaller mirror, the slit will be 100 microns. The slit for the larger mirror will be 20 microns.

The analysis we are challenging would assume that the coarser slit would result in a greater bandwidth for the wavelength being detected. But what has been overlooked is the magnification of a diffraction grating at grazing exodus. The wavelengths are stretched out by the leverage from the increased angle of diffraction, so a wider slit is permissible. In fact, what we show in Figure 11 is that the wider slit has the finer division of wavelengths; and when mapped back to the sky, the finer angular resolution.

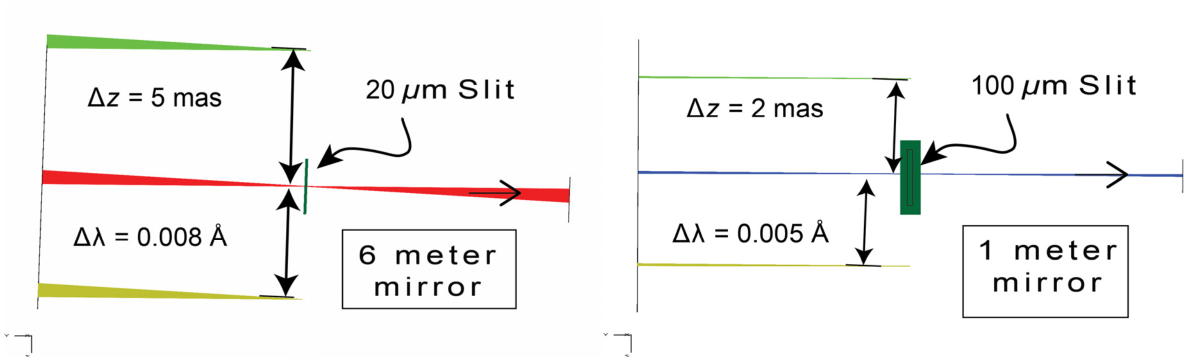

Fig. 11 Although the slit of the 1 m mirror (right) is five times wider than the slit of the 6 m mirror (left), its resolving power is better.

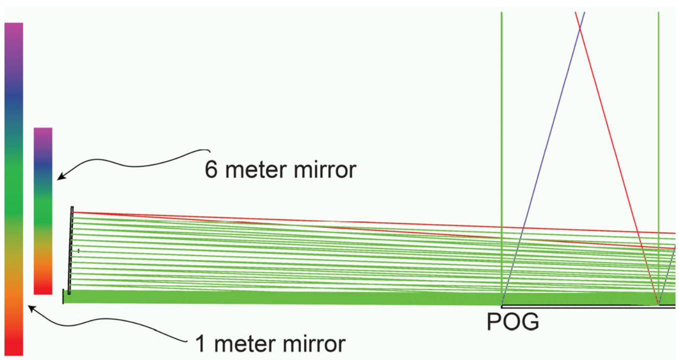

Fig. 12 Conceptual representation of stretched spectra from 2 mirrors

The example is chosen to dramatize. We understand that in the visible light band, slit sizes below 100 µm become spatial filters that produce diffraction artifacts in and of themselves. Moreover, without AO, resolving powers in the milliarcsecond regime cannot be achieved from the ground. The point is that at the theoretical limit of resolution of THE MOST, nothing is lost in angular resolution on the sky as secondary mirrors decrease in size as a result of grazing exodus. One can simply induce from the example of two mirrors that higher spectral and angular resolving powers are preserved over increases in the angle of reconstruction r, notwithstanding that the secondary mirror diameters decrease. Conceptually as shown in Figure 12, the spectrum is stretched at higher angles of reconstruction, permitting lower resolution for the secondary mirror. Indeed, at the effective limit $r = 90°$ there is no space left for any considerable diameter, and the mirror is not parabolic. It is a flat infinitesimal sliver. This is the evanescent case when the light is trapped inside the grating substrate. At this angle of diffraction the POG is at its highest possible resolving power although the reflector has no focusing capability at all.

2.5 Utilization of the POG collection area

The trade that affects THE MOST is not that angular resolving power is traded when smaller secondary mirrors are used at grazing exodus but rather that smaller mirrors limit collection of flux from the very large primary. This limitation goes to Michelson's caveat cited above and must be factored into the discussion of étendue somewhat like the sacrifice of resolving power to gain field-of-view must be factored into a discussion of the triple conjugate LSST. The arithmetic of the AΩ product is insufficient as a figure of merit, although it does hint that THE MOST may be a powerful survey telescope especially with regard to an intrinsic spectrographic capability not found in any of the existing or planned mirror survey telescopes. Revolutionary technologies introduce unfamiliar parameters that trade their undeniable advantages for undesirable limitations.

The ray bundles traced in Figure 11 hint at the problem, since the 10 micron slit appears to be collecting more rays than the wider 50 micron slit. This is because the larger the secondary mirror, the more flux is collected off the POG. The concern is the height of the mirror rather than its width, because it makes no sense for the secondary to be narrower than the POG it collects.. If the mirror is a rectangle, gain in flux collection is linear as a function of the height of the mirror.

A secondary mirror height is affected by the grazing angle. The grazing angle controls the compression of reconstruction of the first-order diffraction beam. The closer to the grating plane, the narrower the beam and hence the smaller the mirror. The function is $d = L \cos(r)$. For fixed r only way to increase d is to increase L. This does not kill the concept but does pose practical problems. The best that can be said is that this is a plane diffraction grating, a surface that can be minted in a continuous roll by embossing plastic. Typical commercial grating rolls from manufacturers are $L > 10$ km.

Flux collection is also controlled by the rate of precession. By way of example, for precession along the equator the transit rate ω_t is 15 arc seconds of angle per second of time. In this case, if the zenith is tagged as time $t = 0$, the angle of incidence i upon the POG can be calculated as $i = t\,\omega_t$. Solving for t in the Grating Equation we have:

$$t = \frac{a\sin\left[\frac{(\lambda - p\sin(r))}{p}\right]}{\omega_t} \tag{4}$$

Integration time over a wavelength band λ_b can then be known by Eq. (5).

$$t_b = \frac{\arcsin\left[\frac{[\lambda_b + \lambda] - p\sin(r)}{p}\right]}{\omega_t} - \frac{\arcsin\left[\frac{\lambda - p\sin(r)}{p}\right]}{\omega_t} \tag{5}$$

In this example, graphed in Figure 13, integration times at the zenith for wavelength bands from 0 to 1 Å width λ_b .

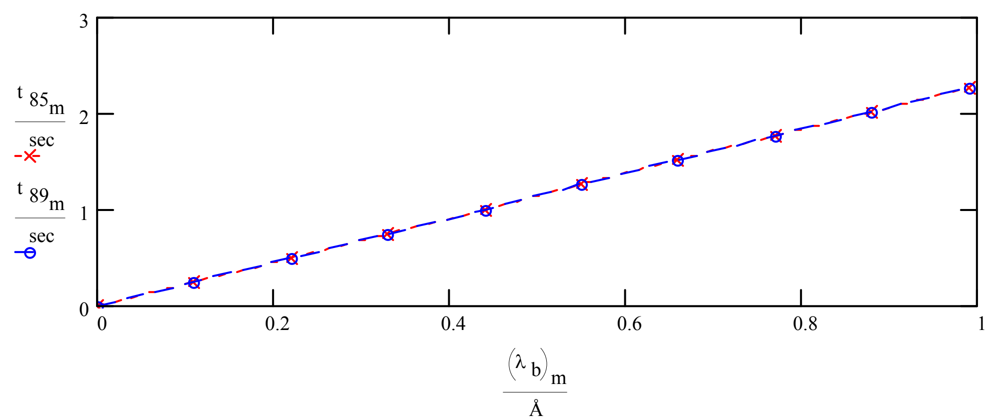

Fig. 13 Integration times for bandwidths $\lambda_b < 1$ Å for $r = 85°$ and $r = 89°$

The grazing exodus configuration produces nearly same rate of precession for angles $r = 85°$ and $r = 89°$. However, the calculation indicates very brief integration periods and relatively coarse spectral bands λ_b . The bands are too broad to acquire a high resolution angle on the sky, and the integrations are too short to acquire faint objects.

We have proposed modifications of the configuration to ameliorate the integration period problem.[6] For example, on the ground integration periods increase at latitudes above the equator. Integration periods increase if the POG is rotated off a strict east/west alignment. Taking advantage of the narrow waist of the ribbon collector, we propose dynamically rotating the POG to track the source. The trade is in bandwidth. In space, precession ω_t is a variable set by the rate of rotation of the telescope and the integration time is as long as the astronomer can wait for a report. Given the tens of millions of objects being surveyed, one would expect a patient audience, but nonetheless this is going to be problematic.

Regardless of variations in deployment that improve performance though increased integration times, there is are variables in the field-of-view and the diameter of the secondary mirror that also must be investigated. The calculation of Figure 13 graphs one slit, but multiple slits can be placed across the image plane. (In practice these might be fibers that can be merged to generate the combined flux across the field-of-view.)

Once the full field-of-view is taken into account, the integration times begin to match those of competing survey scopes such as the LSST which nominally spends 15 seconds on each facet of its mosaic. For THE MOST operating with a field-of-view in the secondary of 1° we see

However, this calculation does not account for the change in magnification taking place at ground level. The secondary mirror diameter can decrease for any grating length as the angle of diffraction grows by $d = L \cos$ (r). Integration time is affected. When these parameters are entered into the equations, we learn that integration time is dependent on the considerable diameter of the secondary mirror. [7]

3. CONCLUSION

This paper was motivated by the assertion that THE MOST was fatally flawed, because there was a misconception that its secondary mirror set the resolving power of the instrument. We have shown that as the chromatic magnification of the POG increases at higher angles of diffraction off the grating surface normal, the allowable slit in front of the secondary spectrograph grows proportionately. The fatal flaw does not exist.

We have also pointed to potential problems with the design that do result from the shrinking size of the secondary mirror diameter as a function of increasing magnification. These problems do not kill the idea, but they are problematic and beg further study.

ACKNOWLEDGEMENTS

David Mozurkewich donated his time to provide a derivation of the conservation of étendue in a POG telescope using the Diffraction Equation and produced his analysis of the relationship of integration time to secondary mirror under a grant to the author from the NASA Institute for Advanced Concepts Research, USRA Sub-award No. 07605-003-060.

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